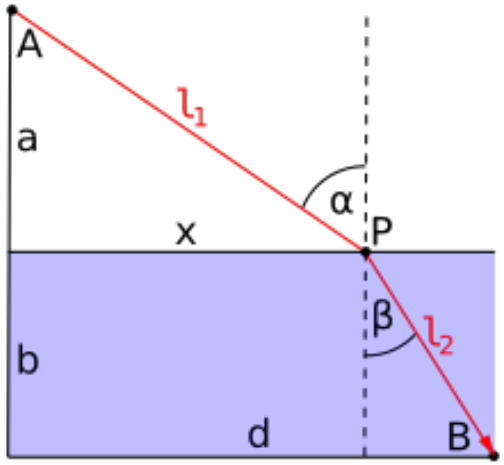
Optical methods in chemistry
or
Photon tools for chemical sciences

Session 9:

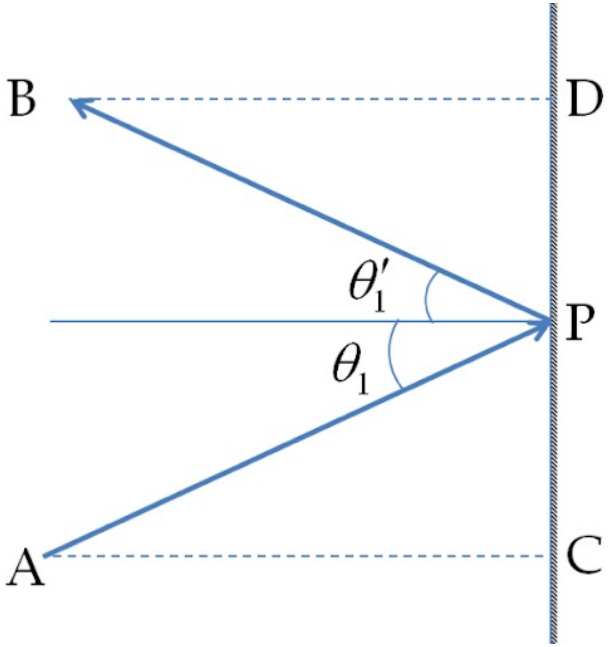
Electromagnetic Waves

Recap: Ray description of light

Fermat Principle

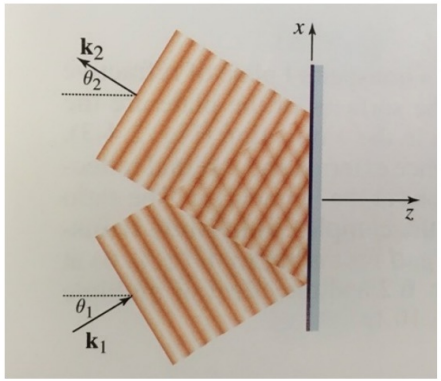


$\mathrm{Min}\{\mathrm{AP}+\mathrm{PB}\} \rightarrow \mathrm{n}_1 \sin\alpha = n_2 \sin\beta$

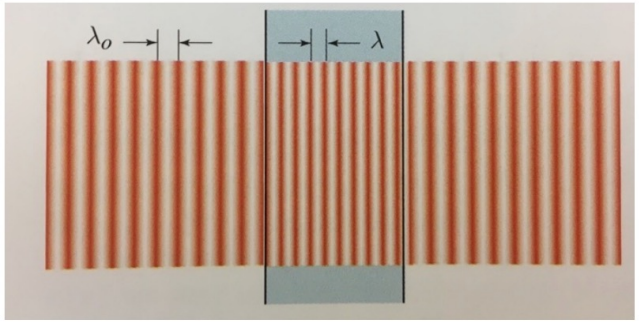


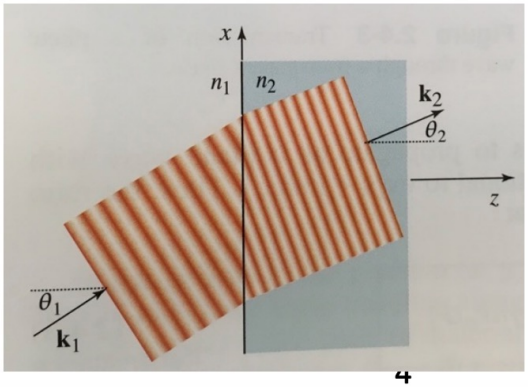
$\mathrm{Min}\{\mathrm{AP}+\mathrm{PB}\} \rightarrow \theta_1 = \theta_1'$

Recap: Wave description of light



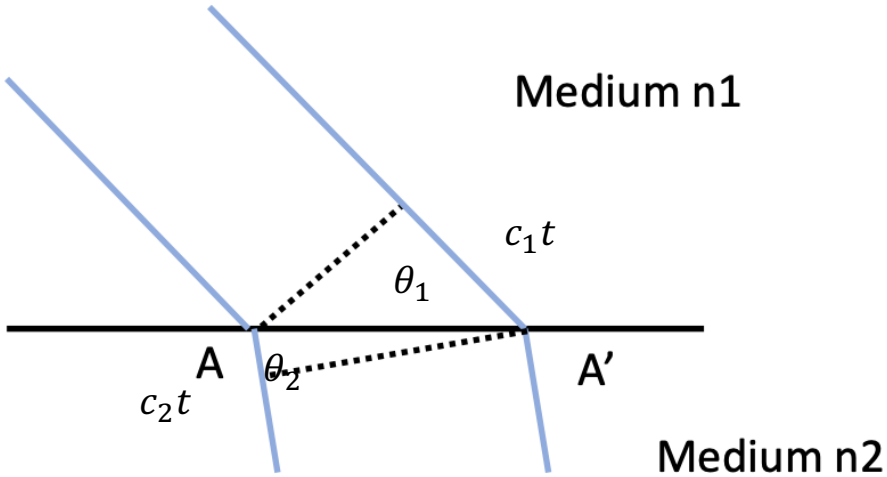
Wavefront: a surface over which the phase of the wave is constant.
Wavelength: period in real space
Wave front is always perpendicular to the propagation direction.



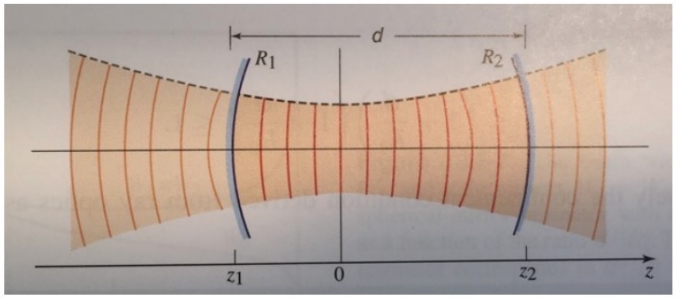


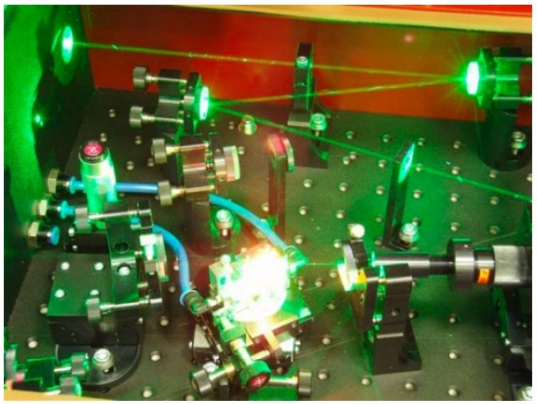
Recap: Wave description of light

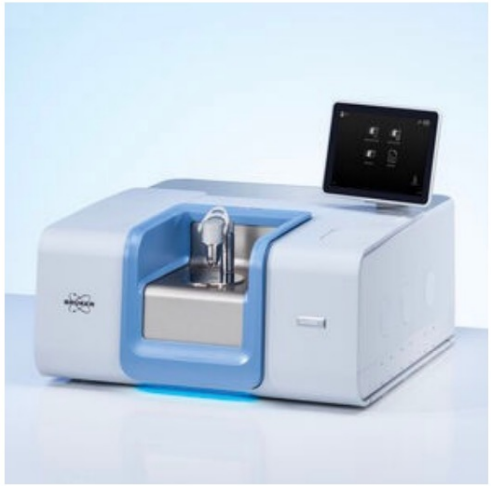
$$\frac{c_1 t}{\cos\theta_1} = \frac{c_2 t}{\cos\theta_2}$$

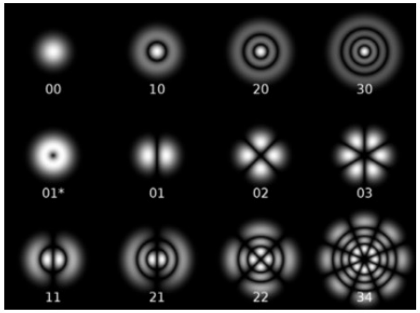


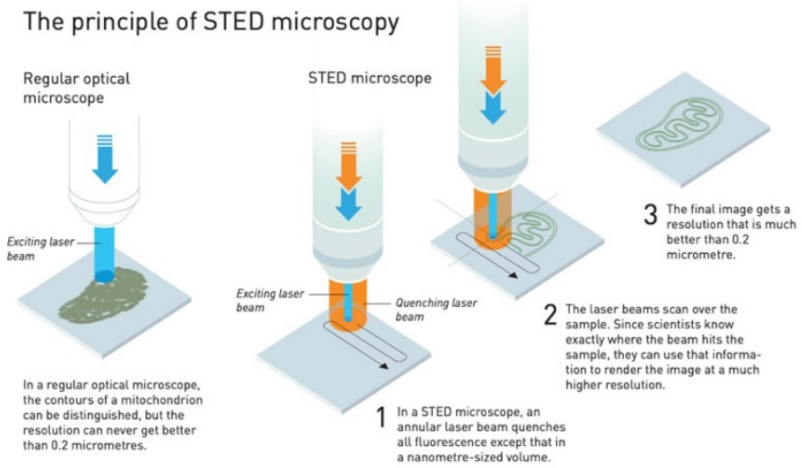
So far we have done well: Fourier optics, beam optics, lasers…





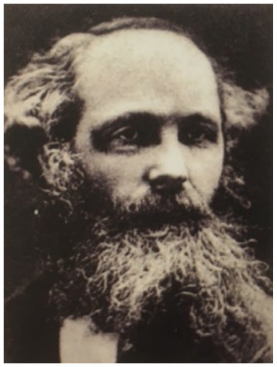






Why beam optics? Isn’t ray optics simple and beautiful?

Welcome to Electromagnetic Wave Description of Light



James Maxwell
1831 - 1879

Maxwell Equations

1. $\nabla \cdot \mathbf{D} = \rho_V$
2. $\nabla \cdot \mathbf{B} = 0$
3. $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
4. $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$

∇:Nabla Operator

$$\nabla = \frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y + \frac{\partial}{\partial z}\vec{e}_z$$

D: Electric Displacement, ρ: Charge Density
B: Magnetic Induction
E: Electric Field
H: Magnetic Field, J: Electric Current Density

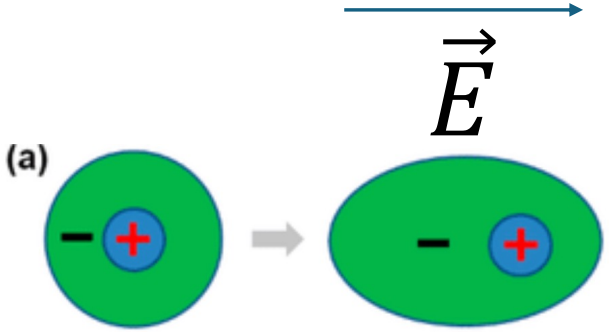
$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

$$\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M}$$

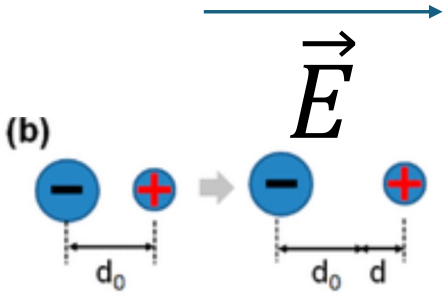
ε_0: Permittivity, electric constant
μ_0: Permeability, magnetic constant

- Four types of Electronic Polarization:
- 1.Free Charge Polarization: Conductive materials
- 2.Ionic Polarization: NaCl crystal
- 3.Orientational Polarization: H2O molecule
- 4.Electronic Polarization: CO2 molecule

- In magnetics, cases are similar, but we won't cover them in this class.



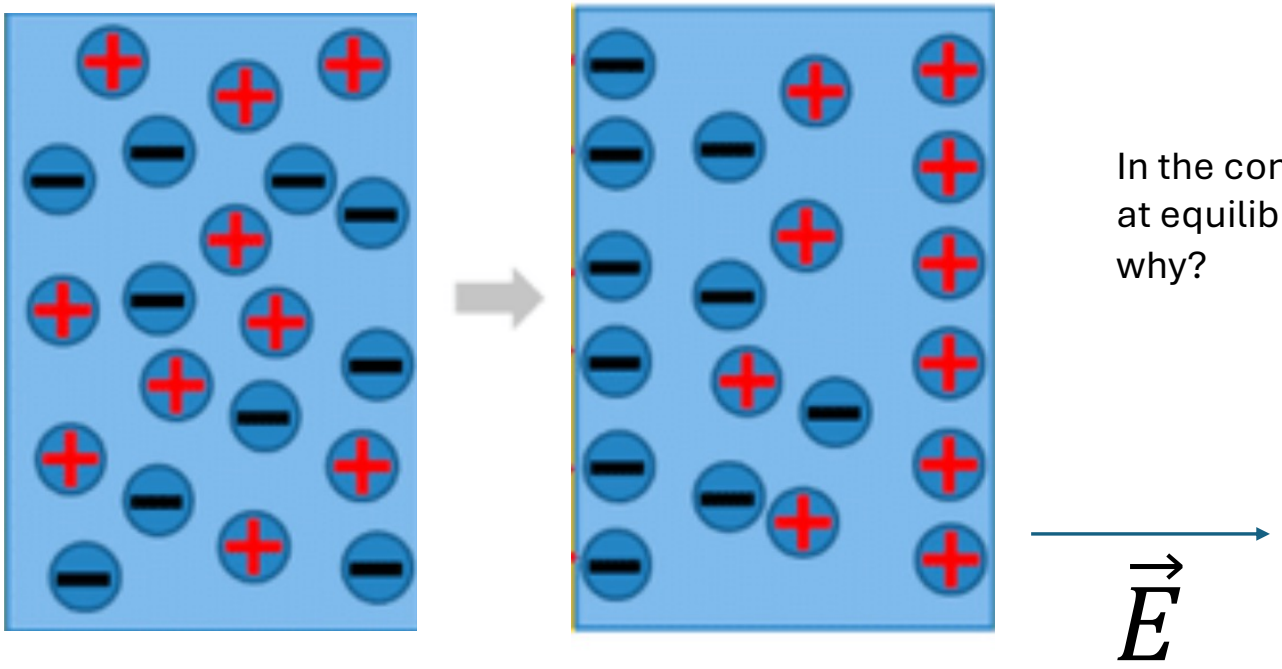
Electronic Polarization



Ionic Polarization

Orientation Polarization

(d)

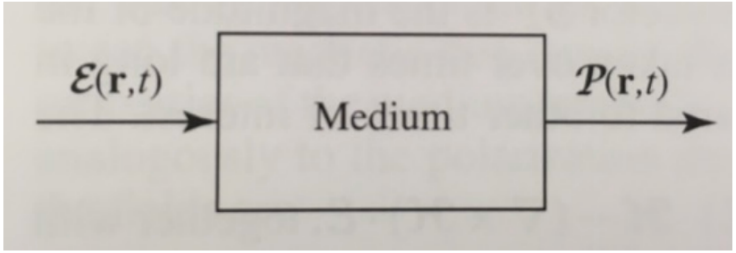


Free Charge Polarization

In the conductive material, at equilibrium state, E = 0, why?

Electromagnetic waves in dielectric media

General



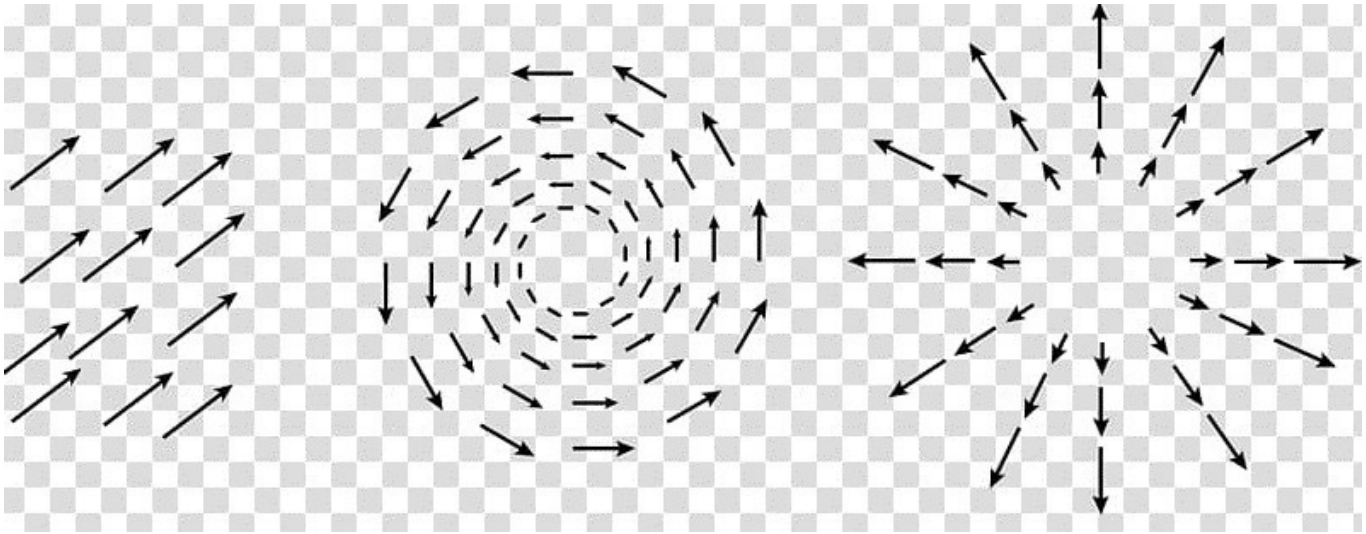
No matter how the polarization is induced, it has the trend to cancel out the original external field.

But stick with linear, nondispersive, homogenous, and isotropic media right now:

$$\mathcal{P} = \epsilon_o \chi \mathcal{E},$$

Divergence and Curl

- Water Flow



Normal Flow Vortex (rotating flow) Water Spring

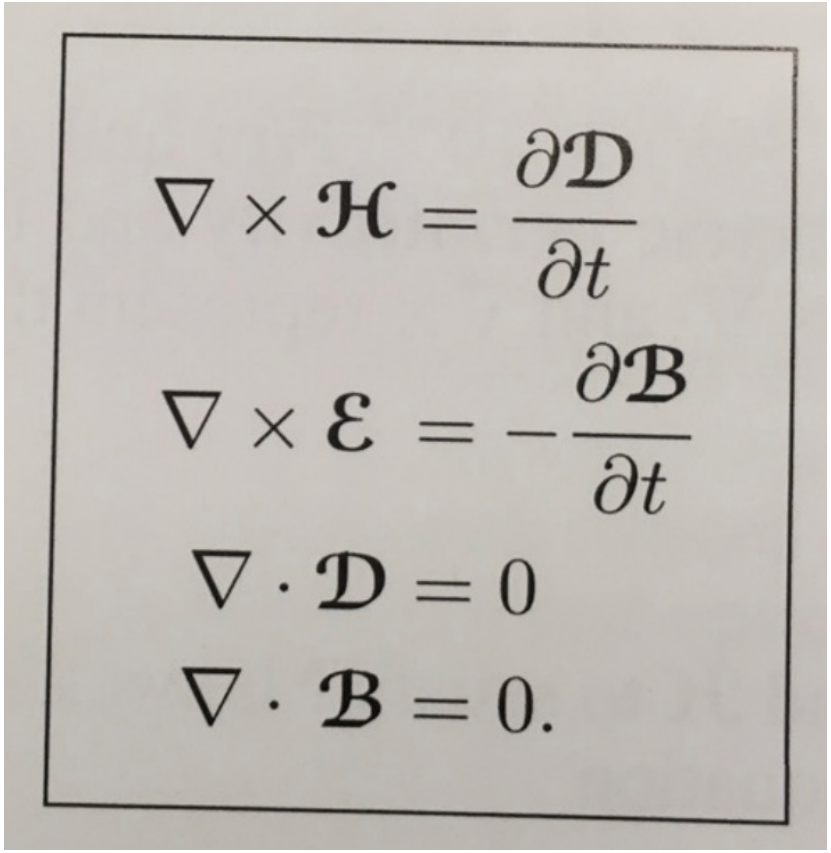
- Water Flow
- Divergence ≠ 0 equivalent to find a small box region, flow in ≠ flow out, so it results in accumulation of water or lost of water.
- Curl ≠ 0, you need an external driven force, because the water won’t rotate by itself.

Maxwell equation in medium

1. $\nabla \cdot \mathbf{D} = \rho_V$
2. $\nabla \cdot \mathbf{B} = 0$
3. $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
4. $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$

$\rho = 0$

$\mathbf{J} = 0$

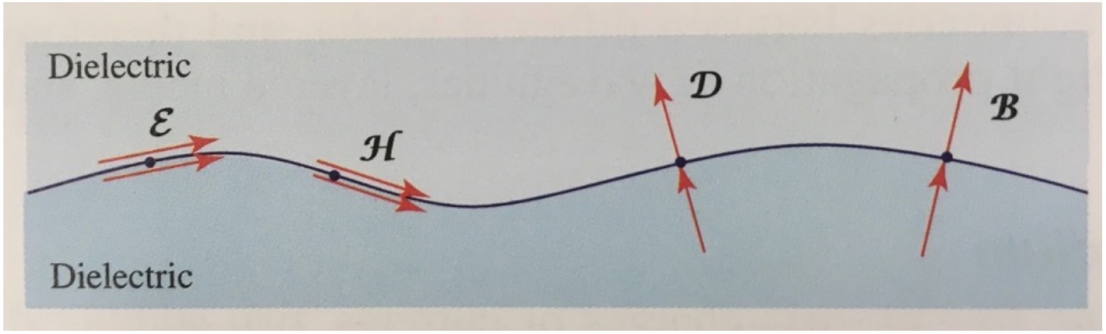


We can safely assume no free charges in the medium, so ρ = 0, no electric current in dielectric medium, so J = 0.

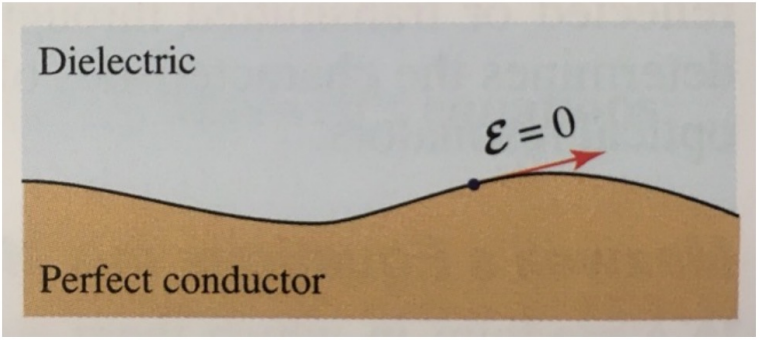
In Equilibrium state, E, H, D, B won't vary with time

Boundary conditions at interfaces

Two dielectric media



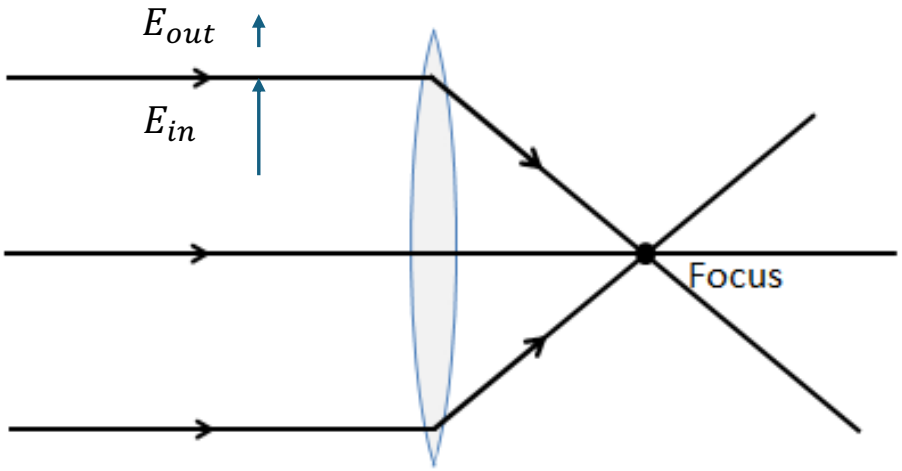
Dielectric and conducting media



1. $\nabla \cdot \mathbf{D} = 0$
2. $\nabla \cdot \mathbf{B} = 0$
3. $\nabla \times \mathbf{E} = 0$
4. $\nabla \times \mathbf{H} = 0$

$$E_{para,in} = E_{para,out}$$
$$H_{para,in} = H_{para,out}$$
$$D_{ortho,in} = D_{ortho,out}$$
$$B_{ortho,in} = B_{ortho,out}$$

Now: What's wrong with Ray Optics (Geometric Optics)?



Geometric Optics Focus

Note: Light (Electromagnetic Wave) is a transverse wave, E and H are perpendicular to propagation direction.

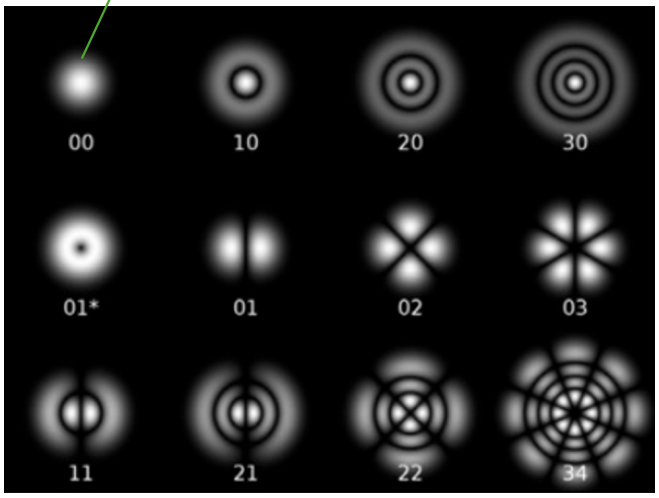
$E_{in} \neq E_{out}$, so $\nabla \cdot \vec{E} \neq 0$, so there has to be charge accumulation at the boundary! In reality, there cannot be charge accumulation in vacuum.

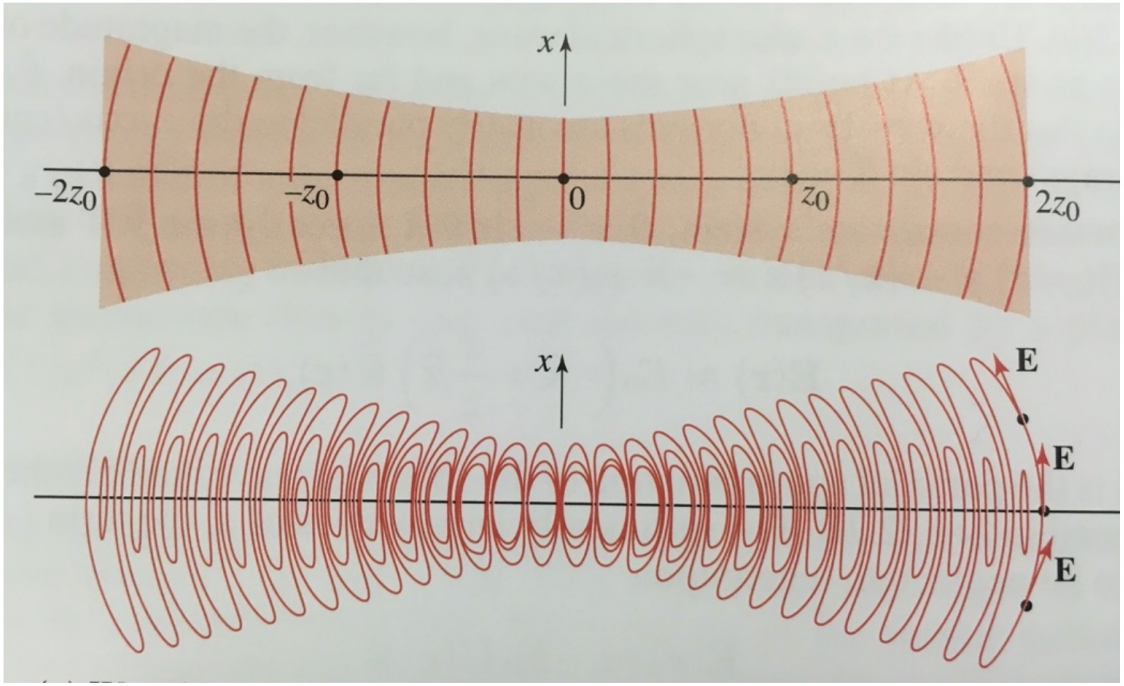
Beam Optics:

Gaussian Beam is one possible solution from Maxwell equations

$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$

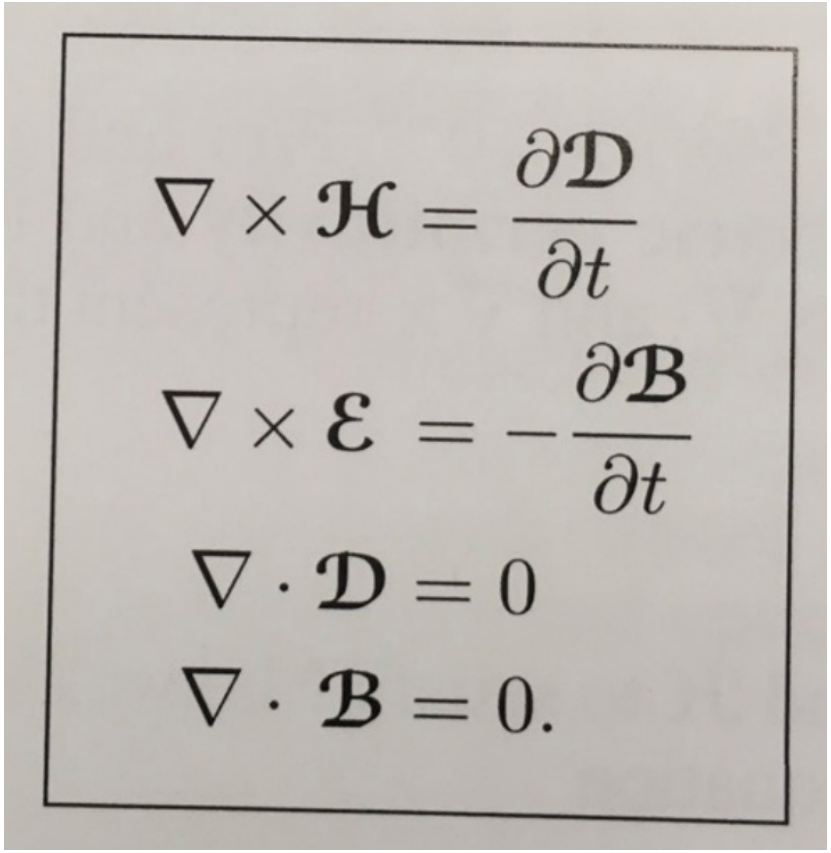
Gaussian Beam





Derive wave equations from Maxwell Equations

Hint: $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$



$$\nabla \times \mathcal{H} = \frac{\partial \mathcal{D}}{\partial t}$$

$$\nabla \times \mathcal{E} = -\frac{\partial \mathcal{B}}{\partial t}$$

$$\nabla \cdot \mathcal{D} = 0$$

$$\nabla \cdot \mathcal{B} = 0.$$

$$\frac{1}{c_0^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = \mathbf{0}$$

$$c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$c = \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_0}} = \frac{1}{n}\frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{c_0}{n}$$

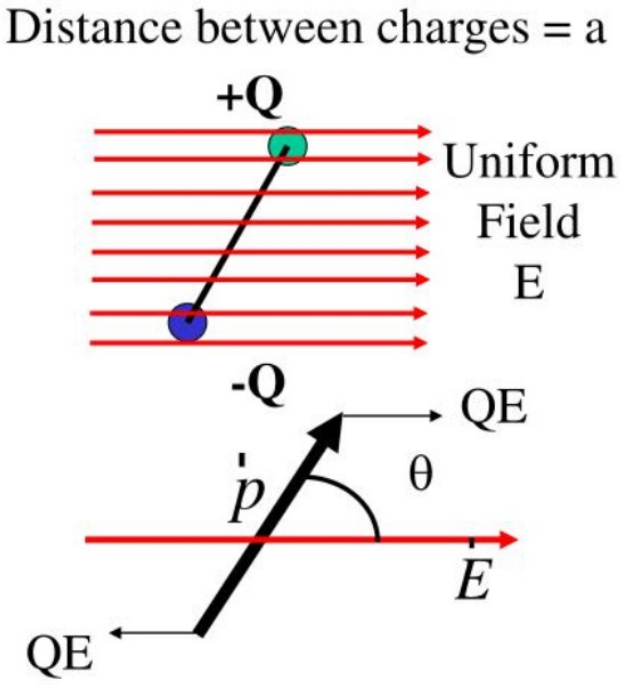
Gradient Force and Optical Tweezer (Optical Trap)

Polarization, electric dipole

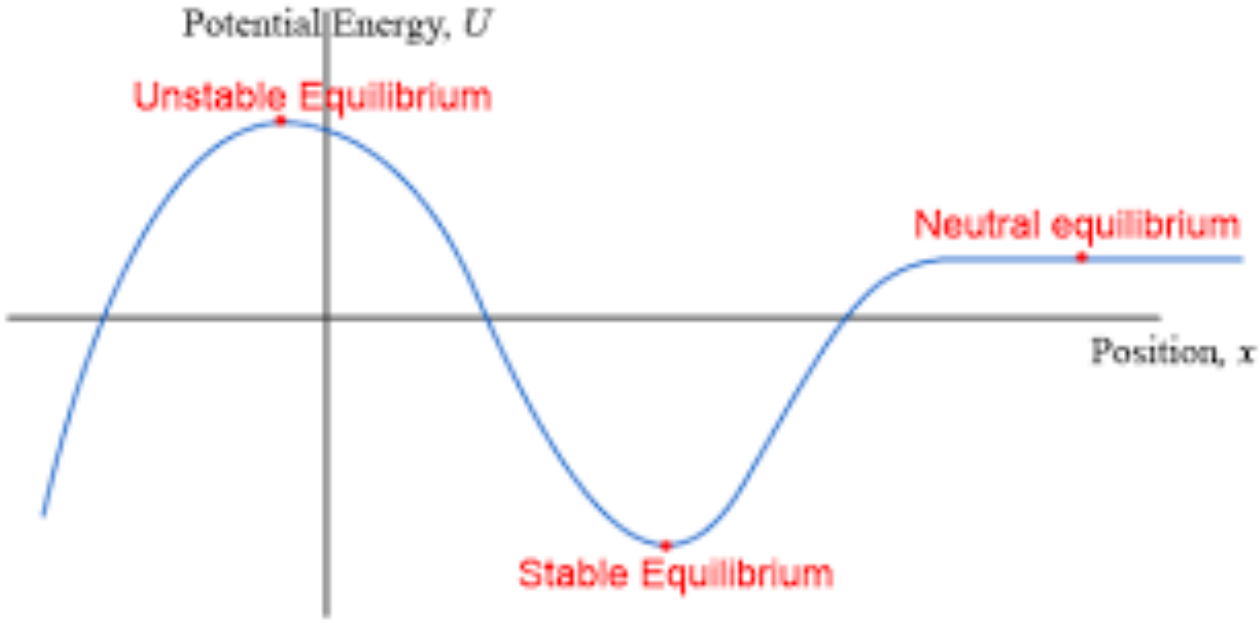
Electronic Energy $= -\vec{E} * \vec{p}$

If we consider the induced dipole of dielectric material, $\vec{p} = \alpha\vec{E}$

Energy = $-\frac{1}{2}\alpha E^2$, $\frac{1}{2}$ is because when the external electric field drive positive charges and negative charges away from each other, half energy goes to kinetic energy and then become heat after the motion stops.



- Minimum Energy Principle
- The system tends to be at an equilibrium position with potential energy at minima.
- Stable Equilibrium, Neutral Equilibrium, Unstable Equilibrium



- Gradient Force

The system has the tendency to evolve from a high energy place to a low energy place (minimum energy principle). That's a basic principle, and that's how we understand where the dynamics will go in energy regime.

We can also see this in Newtonian regime and use force to predict the motion or dynamics.

The two aspects are equivalent. So there must be a force pointing from high energy point to low energy point to guarantee that.

→ gradient force $\mathrm{F_G} = -\nabla \mathrm{E_p}$

Trapping condition in e-m description

288 OPTICS LETTERS / Vol. 11, No. 5 / May 1986

Observation of a single-beam gradient force optical trap for dielectric particles

A. Ashkin, J. M. Dziedzic, J. E. Bjorkholm, and Steven Chu
AT&T Bell Laboratories, Holmdel, New Jersey 07733

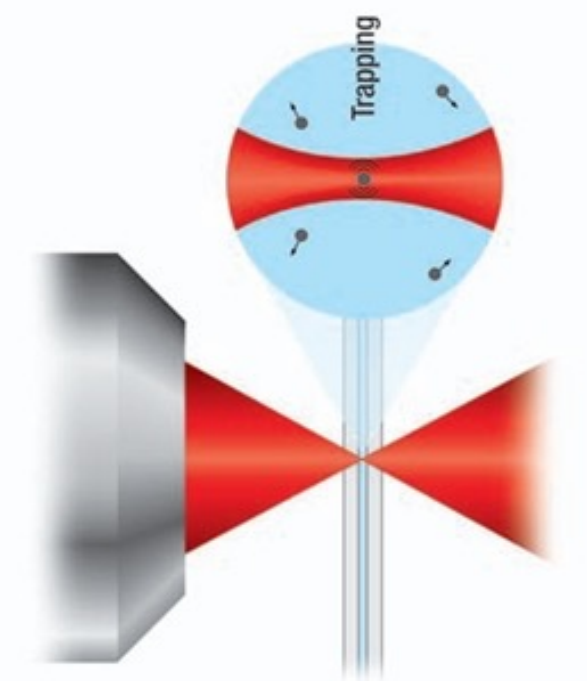
Gradient force

$$\mathbf{F} = \frac{1}{2}\alpha \nabla E^2 = \frac{2\pi n_0 r^3}{c}\left(\frac{m^2-1}{m^2+2}\right)\nabla I(\mathbf{r}) \quad > \quad \mathbf{F}_{\text{scat}}(\mathbf{r}) = \frac{8\pi n_0 k^4 r^6}{3c}\left(\frac{m^2-1}{m^2+2}\right)^2 I(\mathbf{r})\hat{z}$$

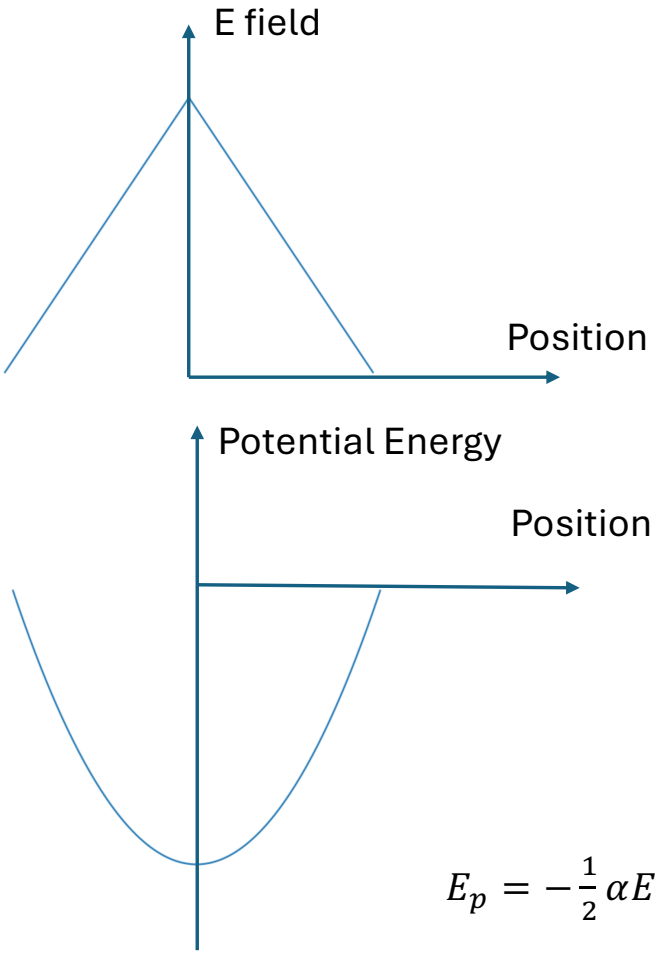
Scattering force

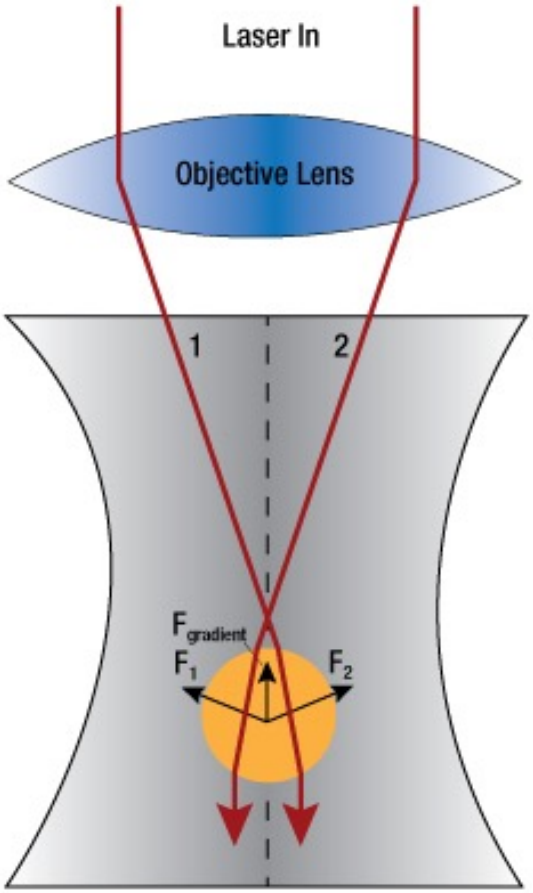
α, induced dipole of sphere
n_o, refractive index and $m=n_1/n_o$ relative index.

Equilibrium Position: Strongest Point



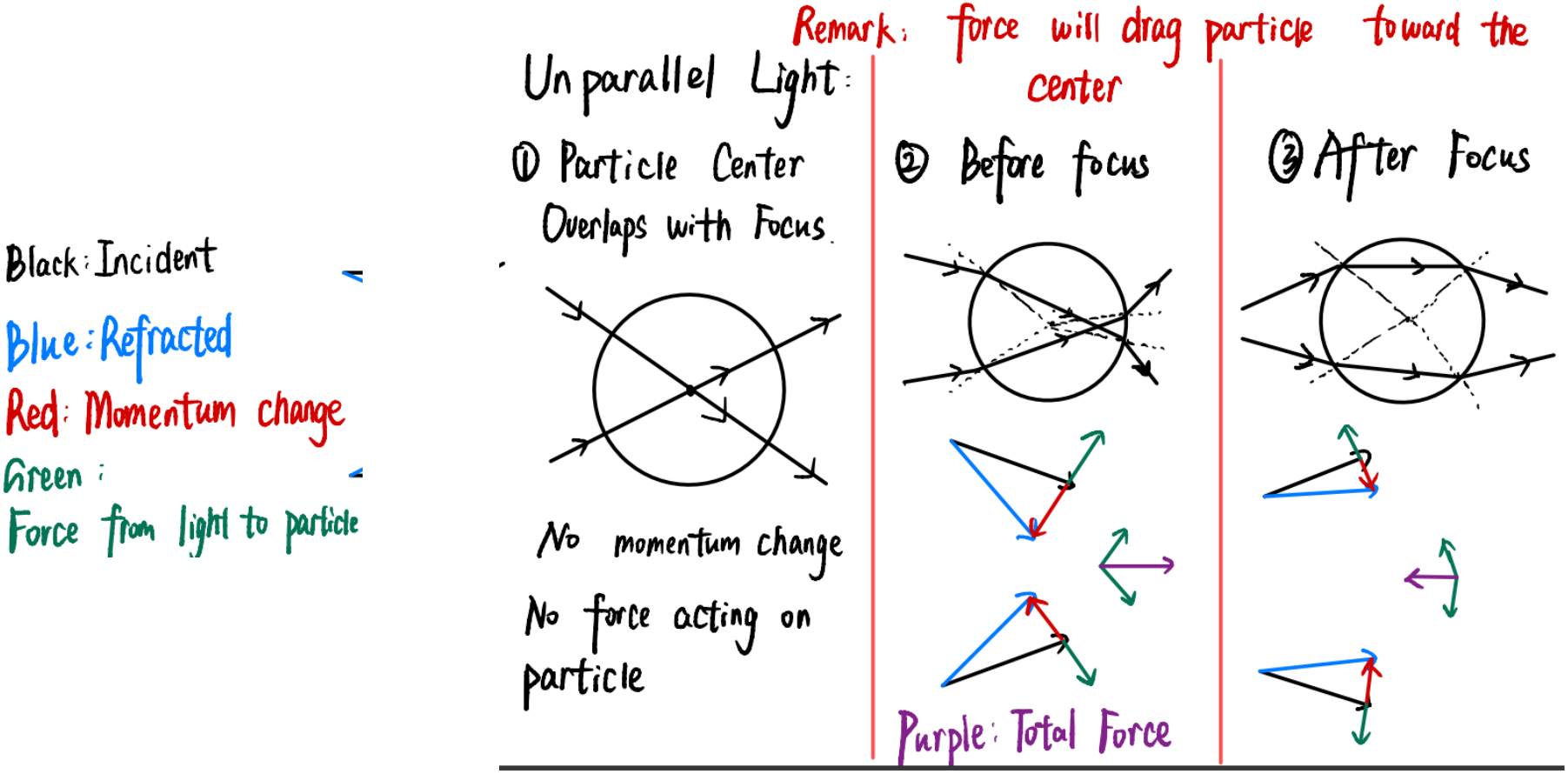
Credits to thor lab





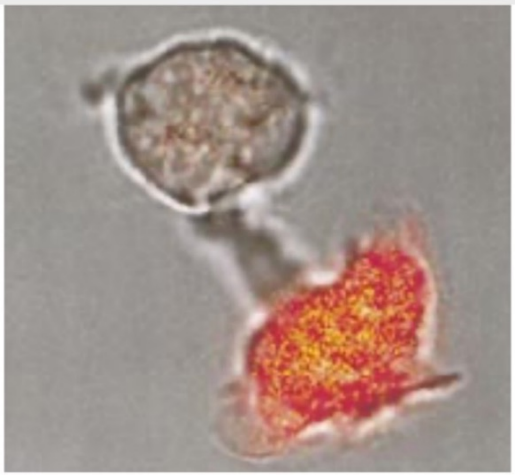
$E_p = -\frac{1}{2}\alpha E^2$, reaches minima at focus

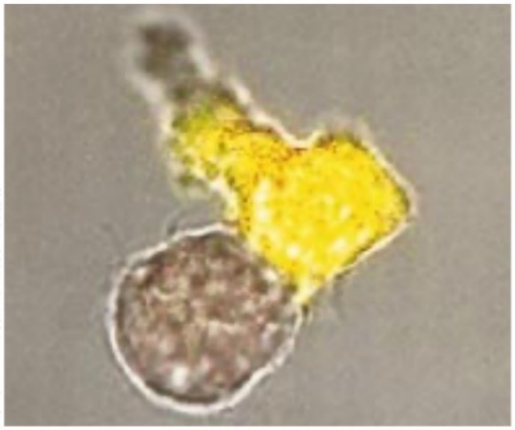
Previous Homework about optical tweezer



We used momentum transfer to analyze the net force in the previous homework, which is Newtonian way of understanding optical trap. And by the way, the net force was gradient force. Now with a well defined potential energy and minimum energy principle, we have a new way of understanding it.

Optical tweezers in biology (example, Scientific American)





XUNBIN WEI *Beckman Laser Institute*

Laser Scissors and Tweezers

Researchers are using lasers to grasp single cells and tinier components in vises of light while delicately altering the held structures. These lasers offer new ways to investigate and manipulate cells

by Michael W. Berns

POLARITY OF *T* CELLS is borne out in studies made possible by laser tweezers. ***B*** cells, which provoke calcium release by ***T*** cells, were carefully positioned alongside ***T*** cells using tweezers. Positioning of the ***B*** cell at one end of a quiescent ***T*** cell elicited no change; a fluorescent red stain in the ***T*** cell remained red (*top*). But when the ***B*** cell touched the other end of the ***T*** cell, calcium was released, signaled by yellow fluorescence (*bottom*).

Ref: Scientific American, April 1998, page 62 onwards

Optical tweezers on the nanoscale

nature nanotechnology

REVIEW ARTICLE

PUBLISHED ONLINE: 7 NOVEMBER 2013 | DOI: 10.1038/NNANO.2013.208

Optical trapping and manipulation of nanostructures

Onofrio M. Maragò[1]*, Philip H. Jones[2], Pietro G. Gucciardi[1], Giovanni Volpe[3] and Andrea C. Ferrari[4]*

Optical trapping and manipulation of micrometre-sized particles was first reported in 1970. Since then, it has been successfully implemented in two size ranges: the subnanometre scale, where light–matter mechanical coupling enables cooling of atoms, ions and molecules, and the micrometre scale, where the momentum transfer resulting from light scattering allows manipulation of microscopic objects such as cells. But it has been difficult to apply these techniques to the intermediate — nanoscale — range that includes structures such as quantum dots, nanowires, nanotubes, graphene and two-dimensional crystals, all of crucial importance for nanomaterials-based applications. Recently, however, several new approaches have been developed and demonstrated for trapping plasmonic nanoparticles, semiconductor nanowires and carbon nanostructures. Here we review the state-of-the-art in optical trapping at the nanoscale, with an emphasis on some of the most promising advances, such as controlled manipulation and assembly of individual and multiple nanostructures, force measurement with femtonewton resolution, and biosensors.